7-34. A particle of mass m slides down a smooth circular wedge of mass M as shown in Figure 7-C. The wedge rests on a smooth horizontal table. Find (**a**) the equation of motion of m and M and (**b**) the reaction of the wedge on m.

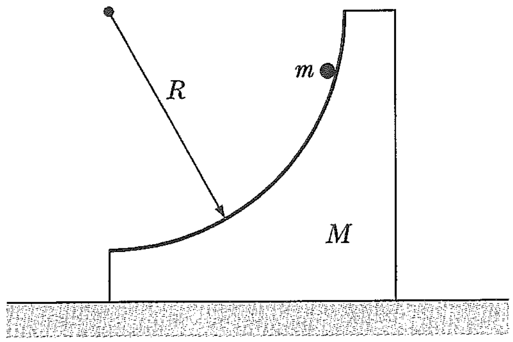

FIGURE 7-C Problem 7-34.

7-35. Four particles are directed upward in a uniform gravitational field with the following initial conditions:

$$\begin{array}{lll} (1) & z(0) = z_0; & p_z(0) = p_0 \\ (2) & z(0) = z_0 + \Delta z_0; & p_z(0) = p_0 \\ (3) & z(0) = z_0; & p_z(0) = p_0 + \Delta p_0 \\ (4) & z(0) = z_0 + \Delta z_0; & p_z(0) = p_0 + \Delta p_0 \end{array}$$

Show by direct calculation that the representative points corresponding to these particles always define an area in phase space equal to $\Delta z_0\, \Delta p_0$. Sketch the phase paths, and show for several times $t > 0$ the shape of the region whose area remains constant.

7.52 ★ The method of Lagrange multipliers works perfectly well with non-Cartesian coordinates. Consider a mass m that hangs from a string, the other end of which is wound several times around a wheel (radius R, moment of inertia I) mounted on a frictionless horizontal axle. Use as coordinates for the mass and the wheel x, the distance fallen by the mass, and ϕ, the angle through which the wheel has turned (both measured from some convenient reference position). Write down the modified Lagrange equations for these two variables and solve them (together with the constraint equation) for $\ddot{x}$ and $\ddot{\phi}$ and the Lagrange multiplier. Write down Newton's second law for the mass and wheel, and use them to check your answers for $\ddot{x}$ and $\ddot{\phi}$. Show that $\lambda \partial f/\partial x$ is indeed the tension force on the mass. Comment on the quantity $\lambda \partial f/\partial \phi$.

7-39. An extremely limber rope of uniform mass density, mass m and total length b lies on a table with a length z hanging over the edge of the table. Only gravity acts on the rope. Find Lagrange's equation of motion.